Exercise 15

A spherical balloon is being inflated. Find the rate of increase of the surface area $(S = 4\pi r^2)$ with respect to the radius r when r is (a) 1 ft, (b) 2 ft, and (c) 3 ft. What conclusion can you make?

Solution

The surface area of a sphere is

$$S(r) = 4\pi r^2.$$

Take the derivative with respect to radius.

$$\frac{dS}{dr} = 8\pi r$$

When the radius is 1 ft, the rate of increase of the surface area is

$$\left.\frac{dS}{dr}\right|_{r=1} = 8\pi(1) = 8\pi \text{ ft}^2 \text{ per foot of radius.}$$

When the radius is 2 ft, the rate of increase of the surface area is

$$\left.\frac{dS}{dr}\right|_{r=2} = 8\pi(2) = 16\pi \text{ ft}^2 \text{ per foot of radius.}$$

When the radius is 3 ft, the rate of increase of the surface area is

$$\left.\frac{dS}{dr}\right|_{r=3} = 8\pi(3) = 24\pi \text{ ft}^2 \text{ per foot of radius.}$$

Notice that the rate of increase of the surface area increases linearly in r.